ADAPTIVE SELF TUNING CONTROL of ROBOT MANIPULATORS with PERIODIC DISTURBANCE ESTIMATION

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Abstract: This paper addresses the problem of position tracking control of robot manipulators in the presence of parametric uncertainty and additive periodic disturbances. Specifically, a self tuning, Lyapunov-based adaptive controller with desired dynamics compensation term and a disturbance estimator has been designed to ensure that the link position tracking error converges to zero asymptotically, despite the partially linearly parametrizable robot dynamics. Extensive experimental results are provided to illustrate the viability and performance of the proposed controller.

Keywords: Robot Manipulators, Model Based Control, Adaptive Self Tuning Control, Disturbance Estimation.

1 Introduction

It is well known that in control practice to achieve better performance, it is imperative to incorporate the system dynamics into the controller design. Robot manipulators are nonlinear, multi-input/multi-output mechatronic systems that have well defined dynamic models; however, exact knowledge of the parameters needed for a model based controller formulation are often not known precisely. In addition, the presence of external disturbances is inevitable under real world conditions. The controller for robot manipulators, therefore have to take account the parametric uncertainties in the dynamics as well as the external additive disturbance effects. These reasons make the robot control problem attractive to numerous researchers. Adaptive controllers provide smooth controller response, therefore unlike its counterparts like sliding mode, robust and variable structure type controllers, do not have the risk of exciting higher order system modes. Due to this benefits, many researchers have preferred adaptive controllers for robot manipulators to compensate for the parametric uncertainty. A detailed review of papers on this topic can be found in [1], and [2]. To name a few, [3] used a linearized version of the robot dynamics and treated the system as a linear plant; however the method proposed was unable to secure the stability persistently. Craig *et al.* [4] was one of the first to propose a nonlinear decoupling adaptive controller based on the computed torque scheme. But the proposed algorithm not only needs to acceleration measurements, but also increases the cost of the computation time due to the inversion of the estimated version of the inertia matrix which is an handicap for the practical applicability. In [5, 6], Slotine *et al.* proposed an inertia related adaptive control law scheme that required position and velocity measurements and did not require the inversion of the estimated inertia matrix. Applying the skew symmetric property between the time derivative of the inertia matrix and the matrix representing the Centripetal and Coriolis effects Slotine *et al* were able to remove the need of acceleration measurements from the controller formulation; however, the transient behavior of the robot manipulator has not considered. Later Sadegh and Horowitz [7] proposed the so called desired compensation adaptation law (DCAL) where the controller was composed of a nonlinear feedforward term consisting a desired regression matrix multiplied by the parameter update vector and an additional nonlinear feedback term to compensate the difference between the actual and the desired regression matrices. This method was further investigated by [8], where the structure of the feedback term were slightly altered and nonlinear damping design tools [9] were utilized to compensate for the difference between the actual and the desired system dynamics. In [10], Johanssan has demonstrated the design of a Lyapunov based adaptive control scheme without using the skew-symmetric property. In order to incorporate the desirable features of using a least-squares update law, Middleton *et al.* [11] augmented the adaptive controller of [4] with additional terms which allowed the closed-loop error system to be written as a stable, linear, strictly-proper, transfer function with the link position tracking error as the output and a prediction error related term as the input. Provided the estimated inertia matrix was forced to be positive-definite (PD) (*i.e.*, a projection-type algorithm was required), this novel input-output relationship facilitated the design of a least squares update law driven by the prediction error while also fostering a bounded-input, bounded-output, global asymptotic link position tracking stability result. Later, the restriction involving the estimated inertia matrix required in [11] was removed in [12], [13], and [14]. In [15], Tang *et al.* developed an adaptive controller which included the standard gradient update law, the composite adaptation update law, and an averaging gradient update law as special cases.

As can be seen from the above paragraph, adaptive controller techniques have been successfully applied to robot manipulators over the last decades. One drawback; however, is that adaptive controllers require the unknown parameters to appear linearly in the system model. Unfortunately, this *linear parameterization* requirement is not applicable to most robot manipulators, due to the friction effects and external disturbances. Most of the adaptive controllers proposed previously have neglected this fact and used a robot model that does not contain the external disturbance terms and/or a precise friction model. To remove this weakness, some researchers have proposed the use of robust and intelligent controller techniques in conjunction with the adaptive controllers. [16, 17] have proposed variable structure based adaptive control methods for robot manipulator that also improve the transient behavior and robustness to the external disturbances. Although the transient behavior has been improved significantly by the proper choice of robust control gains, an acceptable transient response could only be provided by use of relatively high gains which implies high torque signals and would possibly cause saturations of the actuators and chattering on control signal [18]. In recent years, adaptive fuzzy sliding mode controllers has been also adopted to control electrical servo systems such as [19, 20, 21] where the fuzzy system has been used to approximate the unknown nonlinear parts of the controlled process like as a feedback linearization element and the sliding mode control has been used as a feed-forward controller. However those systems are mostly suitable for single-input, single output plants since the designed system becomes too complex and the computational efforts that are needed for fuzzy part highly increases when the system is multi-input, multi-output like a robot manipulator. Similar arguments are also valid for the neural network based adaptive controllers [22].

In this paper, a novel design approach based on a disturbance estimation mechanism in conjunction with a self tuning, DCAL (Desired Compensation Adaptation Law) based approach for the tracking control of robot manipulators that includes parametric uncertainty and additive periodic disturbances is proposed. What makes the proposed method different from the other control schemes is that it does not requires any information of the disturbances acting on the system except for the assumption that it is periodic.

The rest of the paper is organized as follows. Section 2 presents the robot model along with its properties. The control design, error system development, and stability analysis are presented in Sections 3 and 4 respectively. Section 5 contains the experimental results while the conclusions are drawn in Section 6.

2 Robot Model

The dynamic model for an n-link, revolute, direct drive robot manipulator is assumed to be given by the nonlinear ordinary differential equation of the form [23]

$$\mathbf{M}(q)\ddot{q} + \mathbf{C}(q,\dot{q})\dot{q} + \mathbf{G}(q) + \mathbf{F}(\dot{q}) + \xi = \tau, \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ denote the link position, velocity and acceleration vectors respectively, $\mathbf{M}(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, $\mathbf{C}(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal-Coriolis effects, $\mathbf{G}(q) \in \mathbb{R}^n$ is the gravity vector, $\mathbf{F}(\dot{q}) \in \mathbb{R}^n$ represents the friction effects, $\xi \in \mathbb{R}^n$ is a vector containing the unknown but bounded, additive disturbance effects and $\tau(t) \in \mathbb{R}^n$ is the torque input vector. Aside from the well known properties of (1), we will utilize the following properties which will be used in our control development and stability analysis:

Property 1: The inertia matrix $\mathbf{M}(q)$ is symmetric and positive-definite, and satisfies the following inequalities[24]

$$m_1 \|p\|^2 \leq p^T\mathbf{M}(q)p \leq m_2 \|p\|^2 \qquad \forall p \in \mathbb{R}^n \tag{2}$$

where m_1, m_2 are positive constants, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: The left-hand side of (1) can be separated as follows

$$\mathbf{M}(q)\ddot{q} + \mathbf{C}(q,\dot{q})\dot{q} \ + \mathbf{G}(q) + \mathbf{F}(\dot{q}) + \xi = Y\left(q,\dot{q},\ddot{q}\right)\theta + \varsigma, \tag{3}$$

where $Y\left(q,\dot{q},\ddot{q}\right)\theta$ contains the linearly parametrizable (LP) part of (1) and $\varsigma \in \mathbb{R}^n$ contains the rest of the terms that are not LP (such as the additive but periodic disturbance effects). In equation (3) $\theta \in \mathbb{R}^r$ contains some of the constant system parameters (i.e. mass of the links, center of mass of a link, etc.), and the regression matrix $Y(\cdot) \in \mathbb{R}^{n\times r}$ contains known functions dependent on the signals $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$. In our analysis, we will also make use of the common assumption that when the arguments of $Y(\cdot)$ are bounded then $Y(\cdot)$ is bounded [24]. Utilizing (3) we have defined a *desired* version of linear parameterization as

$$\mathbf{M}(q_d)\ddot{q}_d + \mathbf{C}(q_d,\dot{q}_d)\dot{q}_d \ + \mathbf{G}(q_d) + \mathbf{F}(\dot{q}_d) + \xi = Y_d\left(q_d,\dot{q}_d,\ddot{q}_d\right)\theta + \varsigma, \tag{4}$$

similar to that of (3), $Y_d\left(q_d,\dot{q}_d,\ddot{q}_d\right) \in \mathbb{R}^{n\times r}$ is the desired regression matrix where $q_d, \dot{q}_d, \ddot{q}_d \in \mathbb{R}^n$ are the desired joint position, velocity and acceleration profiles respectively. It is also assumed that $q_d(t)$, $\dot{q}_d(t)$, and $\ddot{q}_d(t)$ are all bounded functions of time.

3 Error System Formulation

Our control objective is to design the control torque input signal $\tau(t)$ such that the robots link position vector will approach to any desired position profile as time approaches to infinity (i.e. $q \to q_d$ as $t \to \infty$) despite the presence of uncertain system parameters and periodic disturbance effects. To quantify our control objective the joint space tracking error, denoted by $e(t) \in \mathbb{R}^n$, is defined as follows

$$e = q_d - q. \tag{5}$$

Hence the control objective can now be interpreted as to ensure $e \to 0$ as $t \to \infty$. Due to the presence of parametric uncertainty, our controller is required to contain a means to estimate the system parameters. Therefore we define the difference between the actual and the estimated parameters as

$$\tilde{\theta}(t) = \theta - \hat{\theta}(t), \tag{6}$$

where $\tilde{\theta}(t) \in \mathbb{R}^r$ denotes the parameter estimation error vector and $\hat{\theta}(t) \in \mathbb{R}^r$ denotes the dynamic estimate of the uncertain, constant parameter vector θ defined in (3). To facilitate the subsequent control development and stability analysis, we reduce the order of the robot dynamics given in (1) by defining a filtered tracking error-like variable $r(t) \in \mathbb{R}^n$ as follows

$$r = \dot{e} + \alpha e, \tag{7}$$

where $\alpha \in \mathbb{R}^1$ is a positive constant control gain. In addition, using the periodical disturbance assumption the disturbance term of (3) has been modelled as a time dependent Fourier Series Expansion in the following form

$$\varsigma = E^T Tanh(r) + \sum_{k=1}^{h} D_k^T Cos(kr) + \sum_{k=1}^{h} F_k^T Sin(kr) \text{ with } k = 1, 2... \tag{8}$$

where $E \in \mathbb{R}^{n \times n}$stands for the unknown, mean value of the disturbance terms, $D_k, F_k \in \mathbb{R}^{n \times n}$ with $k = 1, 2...$ representing different error frequencies, are constant matrices with unknown parameters, $h \in \mathbb{Z}_+$ is the harmonic limit for the approximation, and finally the vector functions $Tanh(\gamma)$, $Cos(k\gamma)$, and $Sin(k\gamma)$ are explicitly defined as

$$\begin{aligned} Tanh(\gamma) &\triangleq \begin{bmatrix} \tanh(\gamma_1) & \tanh(\gamma_2) & ... & \tanh(\gamma_n) \end{bmatrix}^T, \\ Cos(k\gamma) &\triangleq \begin{bmatrix} \cos(k\gamma_1) & \cos(k\gamma_2) & ... & \cos(k\gamma_n) \end{bmatrix}^T, \\ Sin(k\gamma) &\triangleq \begin{bmatrix} \sin(k\gamma_1) & \sin(k\gamma_2) & ... & \sin(k\gamma_n) \end{bmatrix}^T. \end{aligned} \tag{9}$$

Remark 1 *Judging from the assumption that the disturbance term is periodic, one might come to conclusion that repetitive learning controllers can also be used when the desired trajectory for the system is also periodic. However, as mentioned in [25] to ensure stability, repetitive learning controllers require the period of all periodic signals to be known, while the method proposed in here does not need this extra assumption.*

Based on the structure of error dynamics, we are motivated to regulate $r(t)$ in order to regulate $e(t)$; hence, we need to calculate the open-loop dynamics for $r(t)$ which is obtained by taking the time derivative of (7), then pre-multiplying the resultant by $M(q)$, and substituting (1) to yield the following advantageous form

$$\mathbf{M}\dot{r} = -\mathbf{C}(q,\dot{q})r + W\theta + E^T\ Tanh(r) + \sum_{k=1}^{h} D_k^T Cos(kr) + \sum_{k=1}^{h} F_k^T Sin(kr) - \tau, \tag{10}$$

where the linear parameterization $W\theta$ is defined explicitly as

$$W\theta \triangleq \mathbf{M}(\ddot{q}_d + \alpha\dot{e}) + \mathbf{C}(q,\dot{q})(\dot{q}_d + \alpha e) + \mathbf{G}(q) + \mathbf{F}\left(\dot{q}\right). \tag{11}$$

4 Control Design and Analysis

Based on the above error system development and the subsequent stability analysis, we design the control torque input $\tau(t)$ as follows

$$\tau = (\hat{K} + k_r)r + e + Y_d\hat{\theta} + \hat{E}^T\ Tanh(r) + \sum_{k=1}^{h} \hat{D}_k^T Cos(kr) + \sum_{k=1}^{h} \hat{F}_k^T Sin(kr), \tag{12}$$

where $\hat{K} \in \mathbb{R}^{n\times n}$ is a positive definite, diagonal, time varying (self tuning adaptable) gain matrix, $k_r \in \mathbb{R}$ is the positive nonlinear damping gain that will assist us for the compensation of the mismatch between the actual and the desired robot dynamics, $Y_d\left(q_d, \dot{q}_d, \ddot{q}_d\right) \in \mathbb{R}^{n\times r}$and $\hat{\theta} \in \mathbb{R}^r$were previously defined in (4), and (6) respectively. $\hat{E} \in \mathbb{R}^{n\times n}$ stands for the estimate of the E, and similarly $\hat{D}_k$ stands for the estimate of D_k, and $\hat{F}_k$ stands for the estimate of F_k. The dynamics of the parameter estimation vector is selected as follows

$$\dot{\hat{\theta}} = \Gamma Y_d^T\left(\cdot\right) r, \tag{13}$$

where $\Gamma \in \mathbb{R}^{r\times r}$ is the diagonal, positive definite parameter adaptation gain matrix. Similarly, the estimates for $\hat{E}$, $\hat{D}_k$, $\hat{F}_k$ are calculated through the following dynamics

$$\begin{aligned} \dot{\hat{E}} &= \Psi\ Tanh(r)r^T, \\ \dot{\hat{D}}_k &= \Psi_k Cos(kr)r^T \qquad k = 1\cdots h \\ \dot{\hat{F}}_k &= \Psi_k Sin(kr)r^T \qquad k = 1\cdots h, \end{aligned} \tag{14}$$

where Ψ and Ψ_k are positive definite diagonal matrices with appropriate dimensions. After substituting (12) into (10), the closed-loop error system for $r(t)$ can be obtained in the following form

$$\mathbf{M}\dot{r} = -\mathbf{C}(q,\dot{q})r + \chi + Y_d\tilde{\theta} - (\hat{K} + k_r)r - e + \tilde{E}^T\ Tanh(r) + \sum_{k=1}^{h} \tilde{D}_k^T Cos(kr) + \sum_{k=1}^{h} \tilde{F}_k^T Sin(kr), \tag{15}$$

where in the above equation the term $Y_d\theta$ have been added and subtracted and the auxiliary term $\chi\left(\cdot\right) \in \mathbb{R}^n$ has been introduced to quantify the difference between the previously introduced linear parameterizations $W\theta$ and $Y_d\theta$ as follows

$$\chi = W\left(\cdot\right)\theta - Y_d\left(\cdot\right)\theta, \tag{16}$$

and the variables $\tilde{E} \triangleq \hat{E} - E$, $\tilde{D}_k \triangleq \hat{D}_k - D_k$, $\tilde{F}_k \triangleq \hat{F}_k - F_k$.

Note that as shown in [24] the norm of the variable χ defined in (16), can be upper bounded in the following form

$$||\chi|| \leq \varrho(x)||x||, \text{with } x = [r^T e^T]^T \in \mathbb{R}^{2n}, \tag{17}$$

where $\varrho(x)$ is a positive bounding function. We now can state the following Theorem

Theorem 1 *The control law described by (12) and the parameter estimation update laws (13) and (14) ensures the global asymptotic convergence of the position tracking error and the filtered tracking error terms defined in (5) and (7) respectively in the sense that*

$$\lim_{t\to\infty} e(t), r(t) = 0 \tag{18}$$

provided that the nonlinear damping gain defined in (12) is selected according to

$$k_r = \alpha + k_n \varrho^2(x) \tag{19}$$

with the nonlinear damping term k_n is selected sufficiently high and the dynamics of the self tuning gain matrix is selected as follows

$$\dot{\hat{K}} = \Omega \hat{K}^{-1} r^T \hat{K} r \;\; \textit{with } \hat{K}(0) \;\; \textit{selected to be positive definite and diagonal} \tag{20}$$

where $\Omega \in \mathbb{R}^{n\times n}$ is the positive definite,diagonal gain update parameter matrix.

Proof: To prove the above result, we define the following non-negative scalar function

$$\begin{aligned} V \;\; = \;\; & \frac{1}{2} r^T \mathbf{M} r + \frac{1}{2} e^T e + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} + \frac{1}{2}\text{Tr}\left\{\hat{K}\Omega^{-1}\hat{K}\right\} + \frac{1}{2}\text{Tr}\left\{\tilde{E}^T\Psi^{-1}\tilde{E}\right\} \\ & + \frac{1}{2}\text{Tr}\left\{\sum_{k=1}^{h}\left(\tilde{D}_k^T\Psi_k^{-1}\tilde{D}_k\right)\right\} + \frac{1}{2}\text{Tr}\left\{\sum_{k=1}^{h}\left(\tilde{F}_k^T\Psi_k^{-1}\tilde{F}_k\right)\right\}, \end{aligned} \tag{21}$$

where the $\text{Tr}\{\cdot\}$ notation is used to denote the trace of a square matrix. Differentiating (21) along the system trajectories, applying the well known skew symmetric relationship between the centripetal Coriolis effects and the time derivative of the inertia matrix and using (7), we obtain,

$$\begin{aligned} \dot{V} \;\; = \;\; & r^T\left\{\chi + Y_d\tilde{\theta} - (\hat{K}+k_r)r - e + \tilde{E}^T Tanh(r) + \sum_{k=1}^{h}\tilde{D}_k^T Cos(kr) + \sum_{k=1}^{h}\tilde{F}_k^T Sin(kr)\right\} \\ & + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + e^T\{-\alpha e + r\} + \text{Tr}\left\{\hat{K}\Omega^{-1}\dot{\hat{K}}\right\} + \text{Tr}\left\{\tilde{E}^T\Psi^{-1}\dot{\tilde{E}}\right\} \\ & + \text{Tr}\left\{\sum_{k=1}^{h}\left(\tilde{D}_k^T\Psi_k^{-1}\dot{\tilde{D}}_k\right)\right\} + \text{Tr}\left\{\sum_{k=1}^{h}\left(\tilde{F}_k^T\Psi_k^{-1}\dot{\tilde{F}}_k\right)\right\}. \end{aligned} \tag{22}$$

Substituting (13) and (14) into (22), using the fact that, the time derivatives of the estimates are the negative of the time derivative of estimation error terms (i.e. $\frac{d}{dt}(\tilde{\cdot}) = -\frac{d}{dt}(\hat{\cdot})$), then cancelling the common terms, and applying the well-known properties of the trace operator allows us to simplify (22) to have the following form

$$\dot{V} = -r^T\{(\hat{K}+k_r)r - \chi\} + e^T\{-\alpha e\} + \text{Tr}\left\{\hat{K}\Omega^{-1}\dot{\hat{K}}\right\}. \tag{23}$$

Inserting (20) into (23) allows us to further simplify (23) as

$$\dot{V} \leq -r^T k_r r + r^T ||\chi|| - \alpha e^T e. \tag{24}$$

At this point applying (17) and (19) an upper bound for the time derivative of (21) can be obtained to have the following form

$$\dot{V} \leq -\alpha ||r||^2 - \alpha ||e||^2 + [\rho ||x|| \, ||r|| - k_{n1}\rho^2 \, \|r\|^2], \tag{25}$$

where $x \in \mathbb{R}^{2n}$ was defined in (17). Adding and subtracting $\frac{\|x\|^2}{4k_n}$ term to the right hand side of (25) yields

$$\dot{V} \leq -\alpha ||r||^2 - \alpha ||e||^2 + \frac{\|x\|^2}{4k_n} - [k_{n1}\rho^2 \, \|r\|^2 - \rho ||x|| \, ||r|| + \frac{\|x\|^2}{4k_n}]. \tag{26}$$

In the above equation the terms in the brackets is square of $\left(\sqrt{k_n}\rho \, |r| - \frac{\|x\|}{2\sqrt{k_n}}\right)$ and due to the negative sign on its front, is always negative, this enables us to further upper bound (26) to have the following the form

$$\dot{V} \leq -\left(\alpha - \frac{1}{4k_n}\right) \|x\|^2 . \tag{27}$$

Finally when the controller gain k_n is selected to satisfy $k_n \gg 4\alpha$, we can conclude that

$$\dot{V} \leq -\lambda \, \|x\|^2 \, , \text{ for some } \lambda > 0. \tag{28}$$

From the structure of (21) and (28) we can conclude that $V\left(t\right) \in \mathcal{L}_\infty$; hence $r, e, \tilde{\theta}, \hat{K}, \tilde{E}, \tilde{D}_k$ and $\tilde{F}_k$ are also bounded functions of time for $k \in \{1, 2, ..., h\}$. Since $r, e \in \mathcal{L}_\infty$, from (7) we can show that $\dot{e} \in \mathcal{L}_\infty$, owing to the boundedness of $q_d\left(t\right)$ and $\dot{q}_d\left(t\right)$, we can also show that the link position and link velocity vector are also bounded (i.e. $q\left(t\right), \dot{q}\left(t\right) \in \mathcal{L}_\infty$). Due to the boundedness of $\hat{K}, \tilde{E}, \tilde{D}_k$, $\tilde{F}_k$ and $\tilde{\theta}$ with $E, D_k, F_k \in \mathbb{R}^{n \times n}$ and $\theta \in \mathbb{R}^r$ being constant variables with proper dimensions, their estimates should also be bounded. From the boundedness of all these signals and application of standard signal chasing arguments, we can conclude that all signals in the closed loop error system, including time derivative of the filtered tracking error term $\dot{r}(t)$ and the control torque input given in (12) are bounded (i.e. $\dot{r}(t), \tau \in \mathcal{L}_\infty$), moreover (28) enables us to show that the signals r, e are square integrable that is $r, e \in \mathcal{L}_2$. With the above information and direct application of *Barbalat's Lemma*[9]. we can conclude that the tracking error term, $e\left(t\right)$, and the filtered tracking error $r\left(t\right)$ terms will approach to zero as time approaches to infinity, as proposed in (18)□

Remark 2 *Using a similar assumption, the disturbance term given in (8) can also be modelled in the following form*

$$\varsigma = E^T Tanh(r) + \sum_{k=1}^{h} D_k^T Cos(k\omega t) + \sum_{k=1}^{h} F_k^T Sin(k\omega t) \text{ with } k = 1, 2... \tag{29}$$

where $\omega \in \mathbb{R}$ being a predefined base frequency term. In this case, with a slight change in the controller and the update rules for $\hat{D}_k$ and $\hat{F}_k$, the same analysis would still be valid. However even when perfect tracking is achieved due to the disturbance estimation terms, the controller will have some output (the same argument is also valid for our original disturbance modelling). The use of $Sin(kr)$ and $Cos\left(kr\right)$ terms in our method is to ensure the removal of some of the disturbance estimation terms (more precisely, all of the sine terms) when perfect tracking is achieved. This also saved us from guessing the ω term.

5 Experimental Results

Our previous simulation based studies[26] have shown that, a controller with a similar structure (without the DCAL extension) performs considerable well even when a periodic disturbance term is artificially feed to the system, however its our belief that to illustrate the performance of a controller experimental verification in a real world situations is a must. In order to illustrate the effectiveness and performance of the proposed adaptive tracking controller extensive experiments have been conducted on a two link revolute, direct-drive, planar robot manipulator. The links of the manipulator are constructed from aluminium, having link lengths of 16 cm (link 1) and 6.5cm (link2) and both joints are actuated by DC motors, (a SanyoDemci motor equipped with a 4096 counts per revolution encoder for the first link and an Escap motor equipped with a 1024 counts per revolution encoder for the second link) that are controlled through Copley Controls Corp Model 4122 DC motor amplifiers operated in torque control mode. Similar to other 2-link planar robots [27] the dynamics of the robot used in the experimental studies is assumed to have the following form

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2\left(\dot{q}_1 + \dot{q}_2\right) \\ p_3s_2\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix} \tag{30}$$

where p_1 (kg·m^2), p_2(kg·m^2), p_3(kg·m^2), are parameters related to the masses of the links and link lengths of the manipulator while f_{d1} and f_{d2} (Nm·sec) are the viscous friction parameters, c_2 and s_2 are used to denote $\cos(q_2)$ and $\sin(q_2)$ respectively. The rest of the friction related and disturbances terms are represented in $\varsigma = \begin{bmatrix} \varsigma_1 & \varsigma_2 \end{bmatrix}^T$. For this dynamic model the unknown parameter vector given in (3) is defined as follows

$$\theta = [p_1,\, p_2,\, p_3,\, f_{d1},\, f_{d2}]^T \tag{31}$$

A Pentium4 3.0 GHz PC operating under Windows XP OS hosts the control algorithm, which was implemented via Wincon 4.0 interface to facilitate real-time graphing, data logging, and the ability to adjust control gains without recompiling the program . Data acquisition and control implementation were performed at a frequency of 1.0kHz using the Quanser Q8 I/O board. As suggested in (8), the estimation of the disturbance terms are obtained

$$\hat{\varsigma} = \hat{E}^T Tanh(r) + \sum_{k=1}^{h} \hat{D}_k^T Cos(kr) + \sum_{k=1}^{h} \hat{F}_k^T Sin(kr) \tag{32}$$

while the estimates $\hat{E}$, $\hat{D}_k$, $\hat{F}_k$ with $k = 1..h$ are calculated via (14). Three set of experiments with three different levels of estimations (for $h = 1, 2$ and 3) for the disturbance terms are performed. For all experiments, the desired position trajectories for links 1 and 2 were selected as follows

$$q_{d1} = 0.7\sin(t)\left(1 - e^{-0.3t^3}\right) \text{ rad} \qquad q_{d2} = 1.2\sin(t)\left(1 - e^{-0.3t^3}\right) \text{ rad} \tag{33}$$

while the actual link positions, velocities, and parameter estimates were initialized to zero. On the first set of Experiments (Experiment #1), the parameter h was set to 1 and the controller gains were tuned until the error performance was similar to that of a conventional PID controller, with all of the initial adaptive estimates set to zero. The controller and adaptation gains were recorded as shown below

$$\begin{array}{c} \alpha = diag\left\{ \begin{array}{cc} 3 & 3 \end{array} \right\},\ \Omega = diag\left\{ \begin{array}{cc} 60 & 0.005 \end{array} \right\}, \\ \Gamma = diag\left\{ \begin{array}{ccccc} 0.03 & 0.03 & 0.03 & 0.001 & 0.0005 \end{array} \right\}, \\ \Psi = \Psi_k = diag\left\{ \begin{array}{cc} 0.4 & 0.025 \end{array} \right\} \text{ with } k = 1..h, \\ K \triangleq \hat{K}\left(0\right) + k_r = diag\left\{ \begin{array}{cc} 1 & 0.25 \end{array} \right\}. \end{array} \tag{34}$$

The second and third experiments were performed using the same gains with the parameter h set to 2 and then 3. where $diag\left\{\cdot\right\}$ is used to represent the square matrix having only diagonal elements.

Table 1 provides a comparison of the absolute value of the maximum steady state error and maximum control torque input, and the integral of the square of the link position tracking errors for all three experiments. Fig. 1 illustrates the link position tracking errors for all 3 experiments while parameter estimates of Experiment #1 is given in Fig. 2 (Since parameter estimation plots for all 3 experiments were similar only one of them was selected and presented here). Fig. 3 illustrates the control torque input to the system in Experiment #1, and finally the estimates for the disturbance terms for all 3 experiments are given in Fig. 4. From Table 1 and Figures Fig. 1 and Fig. 4, it is evident that though the total control torque input to the system at remains at comparative values, with the increase of the parameter h, improved transient and better tracking performances are achieved. Also from Fig. 4 it is quite clear that, with the increase of parameter h, better estimations for the disturbance terms are obtained.

Table 1

		$\max\left\{\left\|e_i\left(t\right)\right\|\right\}$ (10-60sec)	$\max\left\{\left\|\tau_i\left(t\right)\right\|\right\}$	$\mathcal{L}_2$ norm of $e_i\left(t\right)$
$h = 1$	Link 1	2.4134	13.5331	376.9875
	Link 2	6.8259	0.2585	1000.54
$h = 2$	Link 1	2.4135	12.9652	368.1515
	Link 2	3.6351	0.2344	324.8998
$h = 3$	Link 1	1.5792	11.3053	251.8756
	Link 2	1.4072	0.3635	92.0166

6 Concluding Remarks

Due to the unstructured nature of the disturbance term, at least with the current level of knowledge, it is not possible to model it perfectly. Therefore a standard adaptive controller, which requires the system uncertainties to be linear in parameters and the number of parameters to estimate a priori , would not be sufficient to compensate the disturbance effects effectively. However if we, somehow, were able to grasp fractions of the disturbance and add these to the feedforward compensation term, we then would be able to enhance the performance of the adaptive controller. With these idea in mind, we used Fourier Series Representation to model "some" portions of the additive disturbance term. To our best knowledge, we are the first to try this method for the disturbance effects and were successfully backed up our method by experimental results. To sum up, in this paper, we proposed a self tuning, DCAL based adaptive controller with disturbance estimation based on Fourier Series Expansion. The controller design is based on a Lyapunov based analysis and guarantees global asymptotic link tracking. Experimental results illustrating the performance and effect of high order estimates for the disturbance term were also presented.

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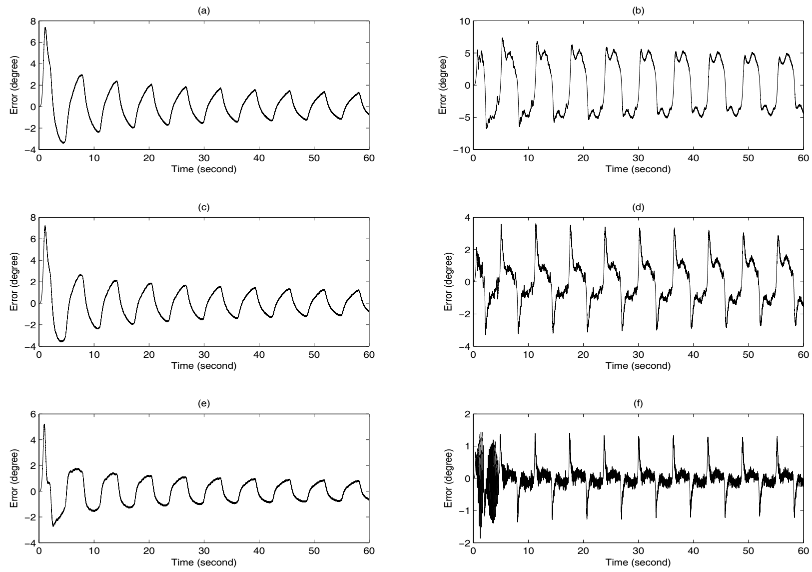

Figure 1: Position Tracking Errors. (a)-(b) are the Error terms for the first and Second Links in the First Experiment while (c)-(d) and (e)-(f) are the Error Terms Recorded during the Second and Third Experiments

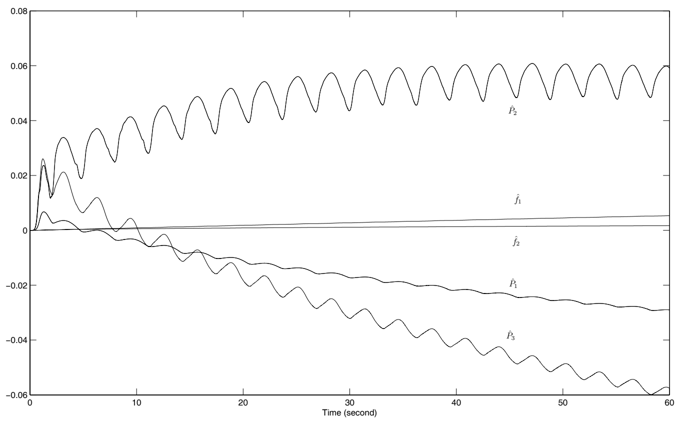

Figure 2: Dynamical Parameter Estimates

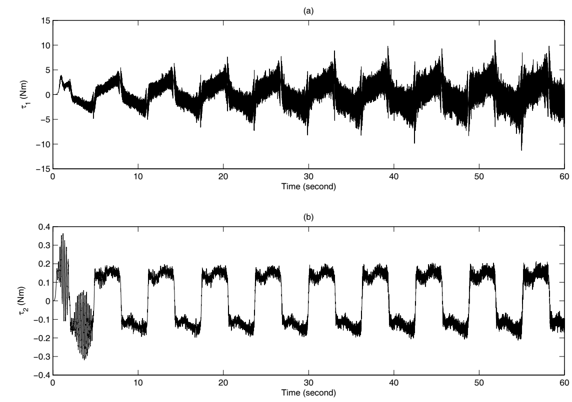

Figure 3: Control Torque Inputs

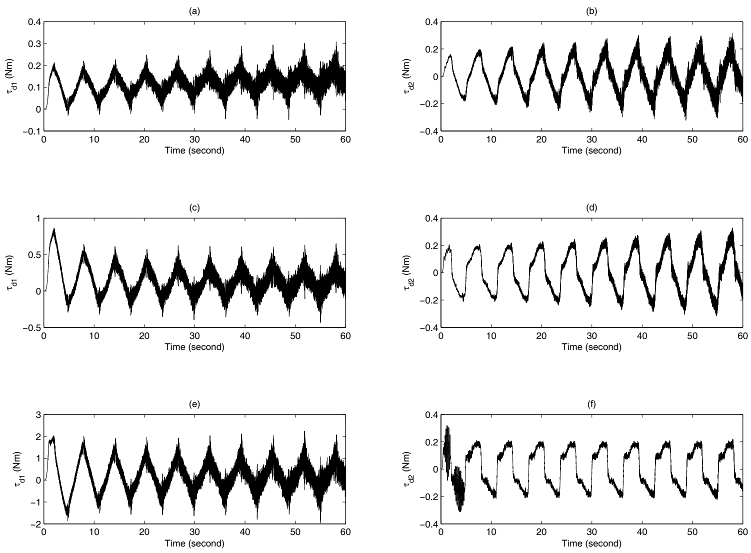

Figure 4: Estimations of the Disturbance Terms of (32). (a)-(b) are Disturbance Estimations for the First Experiment while (c)-(d) and (e)-(f) are the Estimations Recorded for the Second and Third Experiments