1) Consider the following two subsystems, both of which are observable and controllable: \[ \dot{x}_1(t) = \alpha_1 x_1 + u(t), \dot{x}_2(t) = \alpha_2 x_2 + u(t) \] . If these two systems are connected in parallel, such that the output \[ y(t) = x_1 - x_2 \], what are the conditions required for the overall system to be both observable and controllable?

2) If the matrix \( A \) is \( n \times n \), and the matrix \( B \) is \( n \times 1 \), what is (at most) the rank of \[ \Theta_C = \begin{bmatrix} B & AB & A^2 B & \ldots & A^n B & A^{n+1} B & \ldots & A^{2n} B \end{bmatrix} \]

Justify your conclusion by using Cayley-Hamilton Theorem and the Cayley Hamilton Technique.

3) Find \( y(t) \) for
\[ x(t) = \begin{bmatrix} 0 & 1 \\ 8 & -2 \end{bmatrix} x(t), \quad x(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \]
\[ y(t) = \begin{bmatrix} 4 & 1 \end{bmatrix} x(t) \]