1. Let on $S_8$ permutations

\[
A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}, \\
B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}, \\
C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}
\]

a) Write $A, B$ and $C$ as a product of disjoint cycles.
b) Write $A, B$ and $C$ as a product of transpositions.

2. Express on $S_8$ the permutation

\[
\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 8 & 5 & 6 & 4 & 7 & 1 \end{pmatrix}
\]

a) Write as a product of disjoint cycles.
b) Find the order of $\sigma$.
c) Write as a product of transpositions.
d) Is $\sigma$ even or odd permutation.
e) Show that the inverse of the permutation $\sigma$.

3. Express on $S_6$ the following permutations as

\[
A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 6 & 4 & 5 \end{pmatrix}
\]

a) Write as a product of disjoint cycles.
b) Find the order of $A$ and $B$.
c) Write as a product of transpositions for $A$ and $B$.
d) Are $A$ and $B$ even or odd permutation.
e) Show that the inverse of the permutation $\sigma$.
f) $AB = ?$, $BA = ?$ and $AB \neq BA$.
g) $o(AB) = ?$ and $o(BA) = ?$
h) Find the inverse of $AB$.

4. Find the order

a) $(123) \circ (45)$ in $S_5$.
b) $(1234) \circ (56)$ in $S_6$.
c) $(1234) \circ (567)$ in $S_7$. 

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5. a) Let \( \alpha = (13) \circ (58) \) and \( B = (2367) \in S_8 \). Find \( \alpha \circ \beta \circ \alpha^{-1} \) and order of \( \alpha \circ \beta \circ \alpha^{-1} \).

   b) Let \( \alpha = (259) \circ (136) \) and \( B = (157) \circ (2469) \) on \( S_8 \). Find \( \alpha \circ \beta \circ \alpha^{-1} \) and order of \( \alpha \circ \beta \circ \alpha^{-1} \).

6. Let permutation \( \pi = (cebir) \circ (burc) \circ (birey) \) in \( S_{29} \).

   a) Write \( \pi \) as a product of disjoint cycles.
   b) Find the order of \( \pi \).
   c) Write \( \pi \) as a product of transpositions
   d) Is \( \pi \) even or odd permutation
   e) Show that the inverse of the permutation \( \pi \).

7. Let permutation \( S_4 \)

   a) Write all elements of \( S_4 \)
   b) \( o (S_4) = ? \)
   c) Determine \( A_4 \)
   d) \( o (A_4) = ? \)
   e) Write all elements of \( A_4 \) as a product of 3-cycles.
   f) Show that \( A_4 \leq S_4 \)
   g) Find the center of \( A_4 \). \( [C(A_4) = ?] \)

8. Let \( GL(2, \mathbb{R}) \) denote the group all nonsingular \( 2 \times 2 \) matrices over \( \mathbb{R} \).

   Show that each of the following sets is a subgroup of \( GL(2, \mathbb{R}) \)

   a) \( S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc = 1 \right\} \)

   b) \( S = \left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} : a \neq 0 \right\} \)

9. Let \( G \) be a group and define as follows \( M_a = \{ g \mid ga = ag, \ g \in G \} \)

   a) Show that the set \( M_a \) is a subgroup of \( G \).
   b) If \( G = Q_8 = \{ \pm 1, \pm i, \pm j, \pm k \mid i^2 = j^2 = k^2 = ijk = -1 \} \) and \( a = k \),

   write the elements of subgroup \( M_k \).
   c) If \( G = Q_8 \) and \( a = -1 \), write the elements of subgroup \( M_{-1} \).

10. a) Find the subgroups of \( \mathbb{Z} \).
    b) Find the subgroups of Klein 4-group.
    c) Find the subgroups of \( Q_8 \).