1. Express on $S_7$ the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 4 & 6 & 7 & 3 \end{pmatrix}$$

a) Write as a product of disjoint cycles.
b) Find the order of $\sigma$.
c) Write as a product of transpositions
d) Is $\sigma$ even or odd permutation
e) Show that the inverse of the permutation $\sigma$.

2. Express on $S_6$ the following permutations as

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 1 & 6 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 5 & 4 & 6 \end{pmatrix}$$

a) Write as a product of disjoint cycles.
b) Find the order of $A$ and $B$.
c) Write as a product of transpositions for $A$ and $B$.
d) Are $A$ and $B$ even or odd permutation
e) Show that the inverse of the permutation $\sigma$.
f) $AB = ?$, $BA = ?$ and $AB \neq BA$.
g) $o(AB) = ?$ and $o(BA) = ?$
h) Find the inverse of $AB$.

3. a) Let $\alpha = (1257)$ and $B = (246) \in S_7$. Find $\alpha \circ \beta \circ \alpha^{-1}$ and order of $\alpha \circ \beta \circ \alpha^{-1}$.

b) Let $\alpha = (1357)$ and $B = (248) \circ (136)$ on $S_8$. Find $\alpha \circ \beta \circ \alpha^{-1}$ and order of $\alpha \circ \beta \circ \alpha^{-1}$.

4. Find the conjugate class of $\beta = (123)$ in $S_3$ and show that two cycles in $S_3$ conjugate iff they have the same length.

5. Let $G$ be the group of $2 \times 2$ matrices under the addition and

$$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + d = 0 \right\}$$. Prove that $H$ is a subgroup of $G$. 

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6. Let $G$ be the group of all non-zero complex numbers $a+bi$ ($a, b$ real, but not both zero) under multiplication, and let

$$H = \{a + bi \in G \mid a^2 + b^2 = 1\}$$

Verify that $H$ is a subgroup of $G$.

7. a) Prove that the intersection of two subgroups of a group $G$ is also a subgroup of $G$.

b) $2\mathbb{Z} \cap 3\mathbb{Z} \subseteq \mathbb{Z}$