22.10.2013 Algebra 1 Homework 4

1. Find the number of generators of a given cyclic group. [By using the Euler-function]
   a) $\mathbb{Z}_{200}$  b) $\mathbb{Z}_{4900}$  c) $\mathbb{Z}_{1001}$

2. a) Find the number of generators of cyclic group $\mathbb{Z}_{18}$ and determine all
generators of $\mathbb{Z}_{18}$.
   b) Find all subgroups of cyclic group $\mathbb{Z}_{36}$ and give their subgroup dia-
gram.

3. Find the order and determine all the elements in the indicated cyclic
group;
   a) The cyclic subgroup of $\mathbb{Z}_{54}$ generated by 30
   b) The cyclic subgroup of $\mathbb{Z}_{30}$ generated by 18

4. Let $G$ is a cyclic group with its order 24
   a) Find the number of generator of $G$
   b) Find the number of subgroups of $G$
   c) Find the subgroup of $G$ generated by 16
   d) Draw their subgroup diagram

5. Let $(\mathbb{Z}_{18}, \cdot)$ multiplicative group
   a) Find the order of this group.
   b) Show whether this group is cyclic?

6. a) Show that $\mathbb{Z}_2 \times \mathbb{Z}_8$ is not a cyclic group.
    b) Show that $\mathbb{Z}_2 \times \mathbb{Z}_{21}$ is not a cyclic group.

7. Let $G$ be must be an abelian group of order $mn$ where $(m,n) = 1$. Assume
   that $G$ contains an element $a$ of order $m$ and element $b$ of order $n$. Prove that
   $G$ is cyclic group with generator $ab$.

8. Prove that a group of order 3 must be cyclic.

9. a) Find a subgroup of $S_5$ that is a cyclic group of order 6.
    b) Prove or disprove if every subgroup of a group $G$ is cyclic, then $G$ is a
    cyclic group.

10. Prove that $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \mid n \in \mathbb{Z} \right\}$ is a cyclic subgroup of $GL(2, \mathbb{R})$. 