DC motors
DC power systems are not very common in the contemporary engineering practice. However, DC motors still have many practical applications, such as automobile, aircraft, and portable electronics, in speed control applications...

An advantage of DC motors is that it is easy to control their speed in a wide diapason.

DC generators are quite rare. Most DC machines are similar to AC machines: i.e. they have AC voltages and current within them. DC machines have DC outputs just because they have a mechanism converting AC voltages to DC voltages at their terminals. This mechanism is called a commutator; therefore, DC machines are also called commutating machines.
The simplest DC machine

The simplest DC rotating machine consists of a single loop of wire rotating about a fixed axis. The magnetic field is supplied by the North and South poles of the magnet. Rotor is the rotating part; Stator is the stationary part.
The simplest DC machine

We notice that the rotor lies in a slot curved in a ferromagnetic stator core, which, together with the rotor core, provides a constant-width air gap between the rotor and stator.

The reluctance of air is much larger than the reluctance of core. Therefore, the magnetic flux must take the shortest path through the air gap.

As a consequence, the magnetic flux is perpendicular to the rotor surface everywhere under the pole faces.

Since the air gap is uniform, the reluctance is constant everywhere under the pole faces. Therefore, magnetic flux density is also constant everywhere under the pole faces.
1. Voltage induced in a rotating loop

If a rotor of a DC machine is rotated, a voltage will be induced…

The loop shown has sides $ab$ and $cd$ perpendicular to the figure plane, $bc$ and $da$ are parallel to it.

The total voltage will be a sum of voltages induced on each segment of the loop.

Voltage on each segment is:

$$ e_{ind} = \left(v \times B\right) \cdot l \quad (5.5.1) $$
The simplest DC machine

1) *ab*: In this segment, the velocity of the wire is tangential to the path of rotation. Under the pole face, velocity \( v \) is perpendicular to the magnetic field \( B \), and the vector product \( v \times B \) points into the page. Therefore, the voltage is

\[
e_{ba} = (v \times B) \cdot l = \begin{cases} vBl & \text{into page – under the pole face} \\ 0 & \text{– beyond the pole edges} \end{cases}
\]  

(5.6.1)

2) *bc*: In this segment, vector product \( v \times B \) is perpendicular to \( l \). Therefore, the voltage is zero.

3) *cd*: In this segment, the velocity of the wire is tangential to the path of rotation. Under the pole face, velocity \( v \) is perpendicular to the magnetic flux density \( B \), and the vector product \( v \times B \) points out of the page. Therefore, the voltage is

\[
e_{dc} = (v \times B) \cdot l = \begin{cases} vBl & \text{out of page – under the pole face} \\ 0 & \text{– beyond the pole edges} \end{cases}
\]  

(5.6.2)

4) *da*: In this segment, vector product \( v \times B \) is perpendicular to \( l \). Therefore, the voltage is zero.
The simplest DC machine

The total induced voltage on the loop is:

\[ e_{\text{tot}} = e_{ba} + e_{cb} + e_{dc} + e_{ad} \]  \hspace{1cm} (5.7.1)

\[ e_{\text{tot}} = \begin{cases} 
2vBl & \text{under the pole faces} \\
0 & \text{beyond the pole edges}
\end{cases} \]  \hspace{1cm} (5.7.2)

When the loop rotates through 180\(^0\), segment \(ab\) is under the north pole face instead of the south pole face. Therefore, the direction of the voltage on the segment reverses but its magnitude remains constant, leading to the total induced voltage to be
The simplest DC machine

The tangential velocity of the loop’s edges is

$$v = r \omega$$  \hspace{1cm} (5.8.1)

where $r$ is the radius from the axis of rotation to the edge of the loop. The total induced voltage:

$$e_{tot} = \begin{cases} 2r \omega Bl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$  \hspace{1cm} (5.8.2)

The rotor is a cylinder with surface area $2\pi rl$. Since there are two poles, the area of the rotor under each pole is $A_p = \pi rl$. Therefore:

$$e_{tot} = \begin{cases} \frac{2}{\pi} A_p B \omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$  \hspace{1cm} (5.8.3)
The simplest DC machine

Assuming that the flux density $B$ is constant everywhere in the air gap under the pole faces, the total flux under each pole is

$$\phi = A_p B$$  \hspace{1cm} (5.9.1)

The total voltage is

$$e_{\text{tot}} = \begin{cases} \frac{2}{\pi} \phi \omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$  \hspace{1cm} (5.9.2)

The voltage generated in any real machine depends on the following factors:

1. The flux inside the machine;
2. The rotation speed of the machine;
3. A constant representing the construction of the machine.
The simplest DC machine

2. Getting DC voltage out of a rotating loop

A voltage out of the loop is alternatively a constant positive value and a constant negative value. One possible way to convert an alternating voltage to a constant voltage is by adding a commutator segment/brush circuitry to the end of the loop. Every time the voltage of the loop switches direction, contacts switch connection.
The simplest DC machine

segments

brushes
The simplest DC machine

3. The induced torque in the rotating loop

Assuming that a battery is connected to the DC machine, the force on a segment of a loop is:

\[ F = i( \mathbf{l} \times \mathbf{B} ) \]  \hspace{1cm} (5.12.1)

And the torque on the segment is

\[ \tau = rF \sin \theta \]  \hspace{1cm} (5.12.2)

Where \( \theta \) is the angle between \( r \) and \( F \). Therefore, the torque is zero when the loop is beyond the pole edges.
The simplest DC machine

When the loop is under the pole faces:

1. Segment \( ab \):
   \[
   F_{ab} = i \left( \mathbf{l} \times \mathbf{B} \right) = ilB
   \]
   \[
   \tau_{ab} = rF \sin \theta = r \left( ilB \right) \sin 90^\circ = rilB \quad \text{ccw}
   \]
   (5.13.1) (5.13.2)

2. Segment \( bc \):
   \[
   F_{ab} = i \left( \mathbf{l} \times \mathbf{B} \right) = 0
   \]
   \[
   \tau_{ab} = rF \sin \theta = 0
   \]
   (5.13.3) (5.13.4)

3. Segment \( cd \):
   \[
   F_{ab} = i \left( \mathbf{l} \times \mathbf{B} \right) = ilB
   \]
   \[
   \tau_{ab} = rF \sin \theta = r \left( ilB \right) \sin 90^\circ = rilB \quad \text{ccw}
   \]
   (5.13.5) (5.13.6)

4. Segment \( da \):
   \[
   F_{ab} = i \left( \mathbf{l} \times \mathbf{B} \right) = 0
   \]
   \[
   \tau_{ab} = rF \sin \theta = 0
   \]
   (5.13.7) (5.13.8)
The simplest DC machine

The resulting total induced torque is

\[ \tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da} \]  

(5.14.1)

\[ \tau_{ind} = \begin{cases} 2rilB & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \]  

(5.14.2)

Since \( A_p \approx \pi rl \) and \( \phi = A_p B \)  

\[ \tau_{ind} = \begin{cases} \frac{2 \phi i}{\pi} & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \]  

(5.14.3)

The torque in any real machine depends on the following factors:
1. The flux inside the machine;
2. The current in the machine;
3. A constant representing the construction of the machine.
Example of a commutator...
Problems with commutation in real DC machines

A two-pole DC machine: initially, the pole flux is uniformly distributed and the magnetic neutral plane is vertical.

The effect of the air gap on the pole flux.

When the load is connected, a current – flowing through the rotor – will generate a magnetic field from the rotor windings.
Problems with commutation in real DC machines

This rotor magnetic field will affect the original magnetic field from the poles. In some places under the poles, both fields will sum together, in other places, they will subtract from each other.

Therefore, the net magnetic field will not be uniform and the neutral plane will be shifted.

In general, the neutral plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor. The amount of the shift depends on the load of the machine.
Problems with commutation in real DC machines

2) Flux weakening.

Most machines operate at flux densities near the saturation point.

At the locations on the pole surfaces where the rotor mmf adds to the pole mmf, only a small increase in flux occurs (due to saturation).

However, at the locations on the pole surfaces where the rotor mmf subtracts from the pole mmf, there is a large decrease in flux.

Therefore, the total average flux under the entire pole face decreases.
Power flow and losses in DC machines

Unfortunately, not all electrical power is converted to mechanical power by a motor and not all mechanical power is converted to electrical power by a generator…

The efficiency of a DC machine is:

\[ \eta = \frac{P_{out}}{P_{in}} \cdot 100\% \]  

or

\[ \eta = \frac{P_{in} - P_{loss}}{P_{in}} \cdot 100\% \]  

(5.36.1)  

(5.36.2)
There are **five** categories of losses occurring in DC machines.

1. **Electrical or copper losses** – the resistive losses in the armature and field windings of the machine.

   **Armature loss:**
   \[ P_A = I_A^2 R_A \]  
   \[(5.37.1)\]

   **Field loss:**
   \[ P_F = I_F^2 R_F \]  
   \[(5.37.2)\]

   Where \( I_A \) and \( I_F \) are armature and field currents and \( R_A \) and \( R_F \) are armature and field (winding) resistances usually measured at normal operating temperature.
2. Brush (drop) losses – the power lost across the contact potential at the brushes of the machine.

\[ P_{BD} = V_{BD} I_A \quad (5.38.1) \]

Where \( I_A \) is the armature current and \( V_{BD} \) is the brush voltage drop. The voltage drop across the set of brushes is approximately constant over a large range of armature currents and it is usually assumed to be about 2 V.

Other losses are exactly the same as in AC machines…
The losses in DC machines

3. Core losses – hysteresis losses and eddy current losses. They vary as $B^2$ (square of flux density) and as $n^{1.5}$ (speed of rotation of the magnetic field).

4. Mechanical losses – losses associated with mechanical effects: friction (friction of the bearings) and windage (friction between the moving parts of the machine and the air inside the casing). These losses vary as the cube of rotation speed $n^3$.

5. Stray (Miscellaneous) losses – losses that cannot be classified in any of the previous categories. They are usually due to inaccuracies in modeling. For many machines, stray losses are assumed as 1% of full load.
On of the most convenient technique to account for power losses in a machine is the power-flow diagram.

For a DC motor:

\[ P_{\text{in}} = V_T I_L \]

\[ E_A I_A = \tau_{\text{ind}} \omega_m \]

\[ I^2 R \text{ losses} \]

\[ \text{Core losses} \]

\[ \text{Mechanical losses} \]

\[ \text{Stray losses} \]

\[ P_{\text{out}} = \tau_{\text{app}} \omega_m \]

Electrical power is input to the machine, and the electrical and brush losses must be subtracted. The remaining power is ideally converted from electrical to mechanical form at the point labeled as \( P_{\text{conv}} \).
The power-flow diagram

The electrical power that is converted is

\[ P_{\text{conv}} = E_A I_A \]  \hspace{1cm} (5.41.1)

And the resulting mechanical power is

\[ P_{\text{conv}} = \tau_{\text{ind}} \omega_m \]  \hspace{1cm} (5.41.2)

After the power is converted to mechanical form, the stray losses, mechanical losses, and core losses are subtracted, and the remaining mechanical power is output to the load.
The armature circuit (the entire rotor structure) is represented by an ideal voltage source $E_A$ and a resistor $R_A$. A battery $V_{\text{brush}}$ in the opposite to a current flow in the machine direction indicates brush voltage drop. The field coils producing the magnetic flux are represented by inductor $L_F$ and resistor $R_F$. The resistor $R_{\text{adj}}$ represents an external variable resistor (sometimes lumped together with the field coil resistance) used to control the amount of current in the field circuit.
Equivalent circuit of a DC motor

Sometimes, when the brush drop voltage is small, it may be left out. Also, some DC motors have more than one field coil…

The internal generated voltage in the machine is

\[ E_A = K \phi \omega \]  

(5.43.1)

The induced torque developed by the machine is

\[ \tau_{ind} = K \phi I_A \]  

(5.43.2)

Here \( K \) is the constant depending on the design of a particular DC machine (number and commutation of rotor coils, etc.) and \( \phi \) is the total flux inside the machine.

Note that for a single rotating loop \( K = \pi/2 \).
Magnetization curve of a DC machine

The internal generated voltage $E_A$ is directly proportional to the flux in the machine and the speed of its rotation. The field current in a DC machine produces a field mmf $\Phi = N_F I_F$, which produces a flux in the machine according to the magnetization curve.

or in terms of internal voltage vs. field current for a given speed.

To get the maximum possible power per weight out of the machine, most motors and generators are operating near the saturation point on the magnetization curve. Therefore, when operating at full load, often a large increase in current $I_F$ may be needed for small increases in the generated voltage $E_A$. 
Motor types: Separately excited and Shunt DC motors

Seperately excited DC motor:
A field circuit is supplied from a separate constant voltage power source.

For the armature circuit of these motors:

\[ V_T = E_A + I_A R_A \]  \tag{5.45.1}
A terminal characteristic of a machine is a plot of the machine’s output quantities vs. each other.

For a motor, the output quantities are shaft torque and speed. Therefore, the terminal characteristic of a motor is its output torque vs. speed.

If the load on the shaft increases, the load torque $\tau_{\text{load}}$ will exceed the induced torque $\tau_{\text{ind}}$, and the motor will slow down. Slowing down the motor will decrease its internal generated voltage (since $E_A = K\phi\omega$), so the armature current increases ($I_A = (V_T - E_A)/R_A$). As the armature current increases, the induced torque in the motor increases (since $\tau_{\text{ind}} = K\phi I_A$), and the induced torque will equal the load torque at a lower speed $\omega$.

$$\omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2} \tau_{\text{ind}}$$ (5.46.1)
Shunt motor: terminal characteristic

Assuming that the terminal voltage and other terms are constant, the motor’s speed vary linearly with torque.

However, if a motor has an armature reaction, flux-weakening reduces the flux when torque increases. Therefore, the motor’s speed will increase. If a shunt (or separately excited) motor has compensating windings, and the motor’s speed and armature current are known for any value of load, it’s possible to calculate the speed for any other value of load.
Motor types: The series DC motor

A series DC motor is a DC motor whose field windings consists of a relatively few turns connected in series with armature circuit. Therefore:

\[ V_T = E_A + I_A (R_A + R_S) \]  

(5.75.1)
The terminal characteristic of a series DC motor is quite different from that of the shunt motor since the flux is directly proportional to the armature current (assuming no saturation). An increase in motor flux causes a decrease in its speed; therefore, a series motor has a dropping torque-speed characteristic.

The induced torque in a series machine is

\[ \tau_{ind} = K \phi I_A \]  \hspace{1cm} (5.76.1)

Since the flux is proportional to the armature current:

\[ \phi = c I_A \]  \hspace{1cm} (5.76.2)

where \( c \) is a proportionality constant. Therefore, the torque is

\[ \tau_{ind} = Kc I_A^2 \]  \hspace{1cm} (5.76.3)

Torque in the motor is proportional to the square of its armature current. Series motors supply the highest torque among the DC motors. Therefore, they are used as car starter motors, elevator motors etc.
Series motor: terminal characteristic

Assuming first that the magnetization curve is linear and no saturation occurs, flux is proportional to the armature current:

\[ \phi = c I_A \]  \hspace{1cm} (5.77.1)

Since the armature current is

\[ I_A = \sqrt{\frac{\tau_{ind}}{Kc}} \]  \hspace{1cm} (5.77.2)

and the armature voltage

\[ E_A = K \phi \omega \]  \hspace{1cm} (5.77.3)

The Kirchhoff’s voltage law would be

\[ V_T = E_A + I_A (R_A + R_S) = K \phi \omega + \sqrt{\frac{\tau_{ind}}{Kc}} (R_A + R_S) \]  \hspace{1cm} (5.77.4)

Since (5.77.1), the torque:

\[ \tau_{ind} = KcI_A^2 = \frac{K}{c} \phi^2 \]  \hspace{1cm} (5.77.5)
Therefore, the flux in the motor is

\[ \phi = \sqrt{\frac{c}{K}} \sqrt{\tau_{\text{ind}}} \]  

(5.78.1)

The voltage equation (5.77.4) then becomes

\[ V_T = K \sqrt{\frac{c}{K}} \sqrt{\tau_{\text{ind}}} \omega + \sqrt{\frac{\tau_{\text{ind}}}{Kc}} (R_A + R_S) \]  

(5.78.2)

which can be solved for the speed:

\[ \omega = \frac{V_T}{\sqrt{Kc} \sqrt{\tau_{\text{ind}}}} - \frac{R_A + R_S}{Kc} \]  

(5.78.3)

The speed of unsaturated series motor inversely proportional to the square root of its torque.
One serious disadvantage of a series motor is that its speed goes to infinity for a zero torque.

In practice, however, torque never goes to zero because of the mechanical, core, and stray losses. Still, if no other loads are attached, the motor will be running fast enough to cause damage.

Steps must be taken to ensure that a series motor always has a load! Therefore, it is not a good idea to connect such motors to loads by a belt or other mechanism that could break.
DC motor efficiency calculations

To estimate the efficiency of a DC motor, the following losses must be determined:

1. Copper losses;
2. Brush drop losses;
3. Mechanical losses;
4. Core losses;
5. Stray losses.

To find the copper losses, we need to know the currents in the motor and two resistances. In practice, the armature resistance can be found by blocking the rotor and a small DC voltage to the armature terminals: such that the armature current will equal to its rated value. The ratio of the applied voltage to the armature current is approximately $R_A$.

The field resistance is determined by supplying the full-rated field voltage to the field circuit and measuring the resulting field current. The field voltage to field current ratio equals to the field resistance.