EXPERIMENT 1: DIFFERENTIAL AMPLIFIERS

List of components:

Transistors: 4xBC237
Resistors: 1x22kΩ, 2x12kΩ, 1x11kΩ, 2x10kΩ, 2x1kΩ

Objectives:

1) To obtain DC quiescent points of differential amplifiers by performing AC and DC analysis, and also to measure and calculate the common mode gain, the differential mode gain and the common mode rejection ratios of the differential amplifiers.

2) To analyze the effect of the current mirror to the gain of the difference amplifier.

1.1 General Information

Differential amplifiers are generally used to increase the differentiation level of the incoming AC signal. Differential amplifiers are especially used as the first stage of the high gain amplifiers because of their various useful characteristics. It is possible to obtain quite stable and drift resistant amplifiers by choosing the characteristics of the transistors as the same (which is obtained by implementing the transistors on the same silicon wafer with the same W/L ratios) due to the symmetric structure of the difference amplifier. It is ideal to use the difference amplifier if the difference of the two signals which both have a large common DC magnitude is intended to be measured. OPAMP circuits consist of cascade connected differential amplifiers. Thus, it is possible to have stable and high gain amplifiers by using the differential amplifier structure.

Figure 1.1 Basic Differential Amplifier

In order to determine the quiescent points of the transistors in Figure 1.1, DC analysis should be done.
1.1.1 DC Analysis

Necessary equations for obtaining output current and output voltage are given below.

\[ V_1 = V_2 = 0 \]  \hspace{1cm} (1.1)
\[ V_{BE} + I_T R_E - V_{EE} = 0 \]  \hspace{1cm} (1.2)
\[ I_T = \frac{V_{EE} - V_{BE}}{R_E} \]  \hspace{1cm} (1.3)
\[ I_E = I_B + I_C = (1 + \beta)I_B \]  \hspace{1cm} (1.4)
\[ I_T = 2I_E \equiv 2I_C \]  \hspace{1cm} (1.5)
\[ I_C = \frac{V_{EE} - V_{BE}}{2R_E} \]  \hspace{1cm} (1.6)

Output voltage:

\[ V_{O1} = V_{O2} = V_{CC} - I_C R_C = V_{CC} - \left(\frac{V_{EE} - V_{BE}}{2R_E}\right) R_C \]  \hspace{1cm} (1.7)
\[ V_{od} = V_{O1} - V_{O2} = 0 \]  \hspace{1cm} (1.8)

While the inputs are “0”, the difference between the output voltage nodes (V_{OD}) should be “0” in an ideal differential amplifier.

1.1.2 AC Analysis

Small signal AC equivalent circuit of the differential amplifier is shown in Figure 1.2.

![Figure 1.2 AC Equivalent Circuit of Basic Differential Amplifier](image)

Since transistors have the same physical characteristics: \( g_{m1} = g_{m2}; r_{\pi1} = r_{\pi2} \)
\[ g_m = \frac{i_c}{V_T} \quad \text{and} \quad r_\pi = \frac{\beta}{g_m} \]  

(1.9)

The difference between input voltages is called as input differential voltage and denoted by \( V_{id} \).

\[ V_{id} = V_1 - V_2 \]  

(1.10)

Average value of input voltages is called as input common voltage and denoted by \( V_{ic} \).

\[ V_{ic} = \frac{V_1 + V_2}{2} \]  

(1.11)

AC Operation of the circuit can be divided into 2 groups.

1- \( V_1 = -V_2 \), this case is called as differential input mode.

2- \( V_1 = V_2 \), this case is called as common input mode.

**1.1.2.a The Case of Differential Input Signals (The Differential Mode)**

\[ V_1 = -V_2 \]  

(1.12)

\[ V_1 = V_a(t) \]  

(1.13)

\[ V_2 = -V_a(t) \]  

(1.14)

In this case \( V_{ic} \) and \( V_{id} \) become as

\[ V_{ic} = \frac{V_1 + V_2}{2} = 0 \]  

(1.15)

\[ V_{id} = V_1 - V_2 = 2V_a(t) \]  

(1.16)

Equivalent circuit of the differential amplifier when the differential input voltages are applied is shown in Figure 1.3.

![Figure 1.3 The Case of Differential Input Signals](image)
For node E:

\[ g_n V_{n1} + g_m V_{n1} + g_m V_{n2} + g_n V_{n2} = G_E V_e \]  \hspace{1cm} (1.17)

\[-\frac{V_{id}}{2} + V_{n1} + V_e = 0 \Rightarrow V_{n1} = \frac{V_{id}}{2} - V_e \]  \hspace{1cm} (1.18)

\[\frac{V_{id}}{2} + V_{n2} + V_e = 0 \Rightarrow V_{n2} = -\frac{V_{id}}{2} - V_e \]  \hspace{1cm} (1.19)

\[ V_{n1} + V_{n2} = -2V_e \]  \hspace{1cm} (1.20)

\[(V_{n1} + V_{n2})(g_n + g_m) = G_E V_e \]  \hspace{1cm} (1.21)

\[(2g_n + 2g_m + G_E) V_e = 0 \]  \hspace{1cm} (1.22)

In order to satisfy (1.22), since \((2g_n + 2g_m + G_E) \neq 0\), \(V_e\) should be zero.

\[ V_{n1} = \frac{V_{id}}{2} - V_e = \frac{V_{id}}{2} \]  \hspace{1cm} (1.23)

\[ V_{n2} = -\frac{V_{id}}{2} - V_e = -\frac{V_{id}}{2} \]  \hspace{1cm} (1.24)

Output voltages of the circuit are given as

\[ V_{o1} = g_m V_{n1} R_C = -g_m R_C \frac{V_{id}}{2} = -\frac{R_C V_{id}}{r_e} \]  \hspace{1cm} (1.25)

\[ V_{o2} = g_m V_{n2} R_C = g_m R_C \frac{V_{id}}{2} = \frac{R_C V_{id}}{r_e} \]  \hspace{1cm} (1.26)

\[ r_e = \frac{V_{TH}}{I_C} \] is the transresistance of the transistors

Since output differential voltage is \(V_{od} = V_1 - V_2\)

\[ V_{od} = -\frac{R_C V_{id}}{r_e} - \frac{R_C V_{id}}{r_e} = -\frac{R_C}{r_e} V_{id} \]

Differential voltage gain (\(A_{dd}\)) is shown below.

\[ A_{dd} = \left. \frac{V_{od}}{V_{id}} \right| = -\frac{R_C}{r_e} \]  \hspace{1cm} (1.27)

1.1.2.b The Case of Common Input Signals (The Common Mode)

\[ V_1 = V_2 \]  \hspace{1cm} (1.28)

\[ V_1 = V_b (t) \]  \hspace{1cm} (1.29)

\[ V_2 = V_b (t) \]  \hspace{1cm} (1.30)
In this case, $V_{ic}$ and $V_{id}$ become

$$V_{ic} = \frac{V_1 + V_2}{2} = V_b(t) \quad (1.31)$$

$$V_{id} = V_1 - V_2 = 0 \quad (1.32)$$

Small signal AC Equivalent circuit of the differential amplifier when the common input voltages are applied is given in Figure 1.4.

![Figure 1.4 The Case of Common Input Signals](image)

Output voltage equations of the circuit are given as

$$V_{o1} = V_{o2} = -\beta i_b R_c = -\frac{\beta R_c V_b(t)}{r_\pi + 2(1+\beta)R_E} = -\frac{\beta R_c V_{ic}}{r_\pi + 2(1+\beta)R_E} \quad (1.35)$$

The common output voltage becomes $V_{oc} = \frac{V_{o1} + V_{o2}}{2} = V_{o1} = V_{o2}$

Common mode gain ($A_{cc}$) is obtained as

$$A_{cc} = \left. \frac{V_{oc}}{V_{ic}} \right|_{V_{io} = 0} = -\frac{\beta R_c}{r_\pi + 2(1+\beta)R_E} \approx -\frac{R_c}{r_c + 2R_E} \quad (1.36)$$

In an ideal differential amplifier, the common mode gain ($A_{cc}$) is zero. The ratio of differential mode gain ($A_{dd}$) to the common mode gain is called as the Common Mode Rejection Ratio (CMRR). CMRR is an important performance criterion of the differential amplifiers.

$$CMRR = \left| \frac{A_{dd}}{A_{cc}} \right| \quad (1.37)$$
As can be seen from (1.37), in order to increase the CMRR, the magnitude of the resistor \( R_E \) should be increased. An increase in \( R_E \) will require a big increase in the magnitude of the supply voltage to keep the same constant \( I_E \) value. In order to avoid this situation current mirrors are employed as a constant current source (Figure 1.5). Since the AC equivalent resistor of the constant current source is very large, the differential amplifier circuit in Figure 1.5 produces a very small common mode gain (practically very close to zero common mode gain). Therefore a very high CMRR can be obtained without any increase in the \( V_{EE} \) voltage.

![Figure 1.5 a.) Differential Amplifier with Resistor \( R_E \) b.) Differential Amplifier with current mirror](image)

**1.2. Preliminary Studies**

1. Revise operating principles and AC equivalent circuits of differential amplifiers in Figure 1.5a and 1.5b.
2. Calculate the value of \( A_{dd} \), \( A_{cc} \) and CMRR by using the formulas given in this tutorial for Figure 1.5a and 1.5b. SPICE simulations can be used for calculations. You will compare your results with your calculations at the end of the lab.

Note: \( A_{cc} \) can be considered as “0” for current mirror structured differential amplifier in the theoretical calculations.
1.3. Experiment

1. Set up the circuit in Figure 1.5a. Connect + and – supply voltages carefully. Be careful about the grounds of the DC supply voltages and the ground of the circuit.

2. Apply differential voltage (a) to the inputs $V_{i1}$ and $V_{i2}$.
   a. $V_{i1}=10mV.$sin$(2\pi.10^3.t)$
   $V_{i2}=0$
   By measuring the output differential voltage ($V_{o1}-V_{o2}$), find $A_{dd}$
   Note: While measuring the differential voltage, 2 node of 1 probe must be connected to the output terminals.

3. Apply common voltage (b) to the inputs $V_{i1}$ and $V_{i2}$.
   b. $V_{i1}=V_{i2}=10mV.$sin$(2\pi.10^3.t)$
   By measuring the output common voltage ($V_{o1}$ or $V_{o2}$), find $A_{cc}$
   Calculate the value of CMRR by using the measured $A_{dd}$ and $A_{cc}$ values and fill the appropriate space in the table given in the results page.

4. Repeat the above steps for the circuit in Figure 1.5.b.

1.4. Questions

1. Why it is important to have high CMRR? Explain.
2. What is the effect of current mirror to $A_{dd}$, $A_{cc}$ and CMRR?
3. In order to obtain the same CMRR value found for the circuit in Figure 1.5.b with the circuit given in Figure 1.5.a, what would be the value of $R_E$ and $V_{EE}$?
1.5 EXPERIMENT 1 Results Page

**Figure 1.6** Differential and common mode outputs for circuit in Figure 1.5a

**Figure 1.7** Differential and common mode outputs for circuit in Figure 1.5b

**Table 1.1 Comparison of gain and CMRR of the differential amplifiers**

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<thead>
<tr>
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<th>Theoretical Calculation</th>
<th>Experimental Result</th>
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<tbody>
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<td>Figure 1.5.a</td>
<td>Figure 1.5.b</td>
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<tr>
<td>$A_d$</td>
<td></td>
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<tr>
<td>$A_c$</td>
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<tr>
<td>CMRR</td>
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